Influence of dispersive soil electromagnetic properties on hand-held TDEM sensors

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ABSTRACT

Although metal detectors remain the workhorses of humanitarian demining, it is well established that the performance of both continuous wave (frequency domain) and pulsed induction (time domain) detectors can be severely compromised by so-called 'soil-effects'. Generally, problem soils reduce the signal-to-noise ratio and increase the false-detection rate. In certain locations, the soil-effect is so severe as to render the detector practically inoperable. The current study is part of an ongoing international effort to establish and quantify the influence of soil electromagnetic properties on the operation of metal detectors and related sensor technologies. In particular, we examine the relative influence of soil electrical conductivity, magnetic susceptibility and associated frequency dependence on the time domain electromagnetic (TDEM) response of pulsed induction metal detectors and related small-scale TDEM sensors.

Key words: TDEM, Electromagnetic, Soil Properties, Demining.

INTRODUCTION

Despite ongoing development of novel and alternative sensor technologies (MacDonald et al. 2003; GICHD 2006), metal detectors remain the primary tools of humanitarian demining. The extremely low metal content of modern landmines, however, requires that detectors operate at correspondingly higher sensitivity with resulting vulnerability to the modulating influence of background soil conditions. Although the adverse influences of certain soil classes are widely recognized and acknowledged by the demining community, there has only recently been a concerted effort to identify and establish the degree to which specific soil electromagnetic properties are responsible. A range of studies, both empirical and theoretical (Guelle 2002; Billings, Pasion and Oldenburg 2003a; Billings et al. 2003b; Borry, Guelle and Lewis 2003; Das 2004, 2006; Bruschini 2004), has identified magnetic viscosity and the related dispersion of the magnetic susceptibility as principal

sources of interference for time domain metal detectors. In particular, these studies have confirmed the long-established finding (Colani and Aitken 1966) that the time domain electromagnetic (TDEM) response for a viscous magnetic soil decays with t^{-1} dependence compared with $t^{-5/2}$ dependence for a non-magnetic soil having finite, frequency-independent electrical conductivity. The result is a relatively sustained and potentially problematic background response.

Given consensus on the key role of magnetic viscosity, however, the relative influence and potential significance of electrical conductivity dispersion and related chargeability are not well understood. In fact, it emerged through an effort to establish guidelines for the prediction of soil influence on metal detector performance (CEN 2008) that there is a significant lack of clarity on the nature and relative magnitude of the associated response. Consequently, the principal aim of the present study is to address these outstanding issues in the context of an integrated and comprehensive assessment of soil electromagnetic influence on pulse induction metal detectors and related small-scale TDEM systems. In particular, we employ a range of well-established theoretical results to investigate

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and characterize the influence of soil electrical conductivity, magnetic susceptibility and associated frequency dependence on the TDEM response of a surface-deployed, horizontal, coincident coil system.

For present purposes, we ignore the obvious and significant complexities of real-world soils and assume a uniform halfspace with specified soil electromagnetic parameters. Our aim here is to characterize and compare the general nature and extent of the potential influence for a representative range of uniform soil electromagnetic conditions.

General Formulation

In general, for source current *I* impressed on a horizontal, single-turn circular coil of radius *a* positioned at height z = -h above a homogeneous, linear and isotropic soil half-space, the resulting electromagnetic field comprises the following orthogonal electric and magnetic field components as developed in Section 4 (equations (4.86)–(4.88)) of Ward and Hohmann (1987)

$$E_{\phi} = -i\pi f \mu_0 a I \int_0^{\infty} \left[e^{-u_0(z+b)} + r_{\text{TE}} e^{u_0(z-b)} \right] \\ \times \frac{\lambda}{u_0} J_1(\lambda a) J_1(\lambda \rho) d\lambda,$$
(1)

$$H_{\rho} = \frac{aI}{2} \int_{0}^{\infty} \left[e^{-u_{0}(z+b)} + r_{\text{TE}} e^{u_{0}(z-b)} \right] \lambda J_{1}(\lambda a) J_{1}(\lambda \rho) d\lambda$$
(2)

and

$$H_{z} = \frac{aI}{2} \int_{0}^{\infty} \left[e^{-u_{0}(z+b)} + r_{\text{TE}} e^{u_{0}(z-b)} \right] \frac{\lambda^{2}}{u_{0}} J_{1}(\lambda a) J_{0}(\lambda \rho) d\lambda.$$
(3)

Here, E_{ϕ} , H_{ρ} and H_z denote the azimuthal electric field, the radial magnetic field and the vertical magnetic field, respectively. $J_0()$ and $J_1()$ denote Bessel functions of the first kind and integer orders 0 and 1, respectively and λ is the Hankel transform variable. The consistent portion of the integrand in brackets comprises an initial term associated with the primary field and a second term due to the soil-related field, where

$$r_{\rm TE} = \frac{\mu_1 \mathbf{u}_0 - \mu_0 \mathbf{u}_1}{\mu_1 \mathbf{u}_0 + \mu_0 \mathbf{u}_1} \tag{4}$$

denotes the effective reflection coefficient, $u_k^2 = \lambda^2 - \gamma_k^2$, and

$$\gamma_k = 2\pi f \left[\mu_k \epsilon_k \left(i \frac{\sigma_k}{2\pi f \epsilon_k} - 1 \right) \right]^{1/2} \approx \left[-i 2\pi f \mu_k \sigma_k \right]^{1/2} \tag{5}$$

is the quasi-static propagation constant (assumes that displacement currents associated with electrical permittivity ϵ are negligible compared with conduction currents in connection with electrical conductivity σ at operating frequency f), for air (k = 0) and for soil (k = 1), respectively. For air, we have $\sigma_0 = 0$, $\mu_0 = 4\pi \times 10^{-7}$ H/m and, thus $u_0 = \lambda$, yielding

$$r_{\rm TE} = \frac{\mu_1 \lambda - \mu_0 u_1}{\mu_1 \lambda + \mu_0 u_1} \tag{6}$$

Soil electrical conductivity and magnetic permeability are, in general, frequency-dependent complex parameters

$$\sigma_1 = \sigma = \sigma' + i\sigma''$$

$$\mu_1 = \mu = \mu' - i\mu''$$
(7)

The frequency-dependent voltage induced in a coaxial, coplanar single-turn receiver coil of radius b follows as the integral of the azimuthal electric field around the circular coil

$$v(f) = \oint \mathbf{E} \cdot d\ell = b \int_0^{2\pi} E_{\phi} d\phi$$
(8)

or, equivalently, as the time rate of change of integrated magnetic flux density through the coil

$$v(f) = -\frac{\partial}{\partial t} \int_{S} \mathbf{B} \cdot d\mathbf{s} = -i2\pi f \mu_0 \int_0^b \int_0^{2\pi} H_z \rho d\phi d\rho.$$
(9)

The result is

$$v(f) = -i2\pi f \mu_0 \pi a b I \int_0^\infty \left[e^{-u_0(z+b)} + r_{\text{TE}} e^{u_0(z-b)} \right] \times J_1 (\lambda a) J_1(\lambda b) d\lambda$$
(10)

and, on setting b = a, z = -h = 0, we have for coincident, surface deployed coils

$$v(f) = -i2\pi f \mu_0 \pi a^2 I \int_0^\infty [1 + r_{\rm TE}] [J_1(\lambda a)]^2 d\lambda.$$
(11)

Finally, the corresponding time-domain step response is obtained by dividing the previous equation by $i2\pi f$ (equivalent to integrating the associated impulse response) and evaluating the Fourier transform with respect to frequency

$$v(t) = -\mu_0 \pi a^2 I \int_0^\infty \int_{-\infty}^\infty [1 + r_{\rm TE}] e^{i2\pi f t} df \left[J_1(\lambda a) \right]^2 d\lambda \quad (12)$$

or, equivalently, by Laplace transformation with substitution $s = i2\pi f$.

In what follows, we review a range of previously known solutions for specific soil property models and provide a comparative assessment of the related influence on the TDEM response for a small-scale, horizontal coincident-coil configuration at the surface of a uniform soil half-space.

Non-magnetic, non-dispersive conductive soil

For reference, we begin with a uniform non-magnetic, nondispersive conductive soil. Assuming $\sigma_1 = \sigma_{dc}$ and $\mu_1 = \mu_0$, we have

$$1 + r_{\text{TE}} = 1 + \frac{\lambda - u_1}{\lambda + u_1} = \frac{2\lambda}{\lambda + u_1}$$
 (13)

and, consequently, from equation (12)

$$v(t) = -\mu_0 \pi a^2 I \int_0^\infty \int_{-\infty}^\infty \frac{2\lambda}{\lambda + u_1} e^{i2\pi f t} df \left[J_1(\lambda a) \right]^2 d\lambda \qquad (14)$$

with $u_1 = [\lambda^2 + i2\pi f \mu_0 \sigma_{dc}]^{1/2}$. The problem was initially treated by Lee and Lewis (1973) and, subsequently, by Raiche and Spies (1981). On evaluating the Laplace transform of $1/(\lambda + \mu_1)$ and redefining the integration variable $\lambda' = a\lambda$, the resulting solution is

$$v(t) = v_1(t) = \frac{-2\mu_0 \sqrt{\pi} a I}{t} S(\tau), \qquad (15)$$

where

$$S(\tau) = \int_0^\infty \left[e^{-\tau\lambda'^2} - \sqrt{\pi\tau}\lambda' \operatorname{erfc}\left(\sqrt{\tau}\lambda'\right) \right] \sqrt{\tau}\lambda' \left[J_1(\lambda') \right]^2 d\lambda'$$
(16)

is an integral function of normalized time $\tau = t/\sigma \mu_0 a^2$, with erfc() denoting the complementary error function. With substitution $r = \tau \lambda'^2$, $S(\tau)$ is readily evaluated numerically using a Gauss-Laguerre quadrature rule. The result is depicted in Fig. 1. It is observed that for $\tau < 0.005$, $S(\tau)$ becomes constant leaving $\nu_1(t) \propto t^{-1}$ and independent of conductivity. In



Figure 1 Function $S(\tau)$ of normalized time $\tau = t/\sigma \mu_0 a^2$, displaying $t^{-3/2}$ late-time dependence (coincident coil radius a = b = 0.1 m).

particular, the early-time asymptote is $v(t) = \mu_0 a I/2t$ (i.e., $\lim_{\tau \to 0} S(\tau) = 1/4\sqrt{\pi}$).

More significantly, for $\tau > 10$, $S(\tau) \propto t^{-3/2}$, yielding $v(t) \propto t^{-5/2}$. The computed response is depicted in Fig. 2 for three values of σ_{dc} . The absolute signal voltage is normalized for arbitrary source current *I* (and by displaying the absolute value of the response, we effectively chart the response for the typical case of a step-off source current). As predicted, transition from t^{-1} to $t^{-5/2}$ dependence occurs at a later time with increasing soil conductivity. Note for $\sigma_{dc} = 0.1$ mS/m, asymptotic early time t^{-1} dependence at approximately $t = 10^{-10} - 10^{-9}$ s.

Employing a power series expansion for the Bessel function in equation (16) and integrating term by term, Raiche and Spies (1981) obtained the following late-time approximation for $S(\tau)$

$$S(\tau) = \left(\frac{1}{4\tau}\right)^{3/2} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{(2k+2)!}{(k+1)! (k+2)! (2k+5)} \left(\frac{1}{4\tau}\right)^k.$$
(17)

Retaining only the first term, for k = 0, one obtains the common late-time ($\tau > 10$) approximation

$$v(t) \approx v_2(t) = -\frac{\mu_0 \sqrt{\pi} a I}{20t} \left(\frac{\sigma \mu_0 a^2}{t}\right)^{3/2}$$
$$= -\frac{\sqrt{\pi} \mu_0^{5/2} \sigma^{3/2} a^4 I}{20} t^{-5/2}.$$
(18)

The foregoing approximation $v_2(t)$ is compared with the previous result $v_1(t)$ in Fig. 3 for $t \ge 10^{-10}$ s. Clearly, for the majority of metal detectors with measurement windows in the range of ten to several hundred microseconds, this late-time approximation appears to be perfectly adequate.

Non-conductive, non-dispersive magnetic soil

For a non-conductive soil $\sigma_1 = 0$, we have $\gamma_1 = 0$, $u_1 = \lambda$ and, consequently,

$$1 + r_{\rm TE} = 1 + \frac{\mu_0 \lambda \left(\mu_r - 1\right)}{\mu_0 \lambda \left(\mu_r + 1\right)} = 1 + \frac{\chi}{\chi + 2}$$
(19)

where, $\mu_r = \mu_1/\mu_0 = \mu_{dc}/\mu_0$ represents the relative magnetic permeability and $\chi = \mu_r - 1$ is the volume-specific magnetic susceptibility. In particular, for a non-dispersive magnetic soil, $\chi = \mu_{dc}/\mu_0 - 1 = \chi_{dc}$ and equation (12) yields

$$v(t) = -\mu_0 \pi a^2 I \left(1 + \frac{\chi_{dc}}{\chi_{dc} + 2} \right) \delta(t) \int_0^\infty \left[J_1 \left(\lambda a \right) \right]^2 d\lambda, \quad (20)$$



Figure 2 TDEM response for a horizontal, coincident-coil system on a soil half-space.

Note t^{-1} early-time and $t^{-5/2}$ late-time dependence. Response depicted for $\chi_{dc} = 0.001$, $\chi_{dc} = 0.01$ and $\sigma_{dc} = 0.1$ S/m (coincident coil radius a = b = 0.1 m).



Figure 3 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v1 – non-magnetic, non-dispersive conductive soil.

v2 - non-magnetic, non-dispersive conductive soil (late-time).

Response depicted for $\sigma_{dc} = 0.001$, $\sigma_{dc} = 0.01$ and $\sigma_{dc} = 0.1$ S/m (coincident coil radius a = b = 0.1 m).

where the Dirac delta function $\delta(t)$ is defined as follows

$$\begin{split} \delta(t) &= 0; t \neq 0\\ \int_{-\infty}^{\infty} \delta(t) dt &= 1. \end{split}$$

As anticipated, the response for a non-conductive, nondispersive magnetic soil is limited to an impulse at the instant of source current termination (t = 0). There is no sustained transient response. In practice, because detection circuitry is designed to sample the response within one or more timegates with an appropriate delay (typically > 10 μ s) from the step-source termination, no signal is registered.

Das (2004, 2006) treated the more general case of coaxial, coplanar coils at height z = -h above the air-soil interface. In general, the corresponding secondary, soil-related response is

$$v_s(t) = -\mu_0 \pi a^2 I\left(\frac{\chi_{dc}}{\chi_{dc}+2}\right) \delta(t) \mathcal{M}(b), \qquad (21)$$



Figure 4 Log-uniform complex magnetic susceptibility model. Normalized real χ'/χ_{dc} and imaginary χ''/χ_{dc} components are charted as functions of normalized frequency $2\pi f \tau_{\chi}$, with log (τ_2 / τ_1) as parameter.

where

$$\mathcal{M}(b) = \int_0^\infty J_1(\lambda a) J_1(\lambda b) e^{-2\lambda b} d\lambda$$
$$= \frac{2}{\pi k \sqrt{ab}} \left[\left(1 - \frac{k^2}{2} \right) K - E \right]$$
(22)

represents a stand-off dependent coupling coefficient, with $k^2 = 4ab/[(a+b)^2+4h^2]$ and with *K* and *E* representing complete elliptic integrals of the first and second kind respectively. For the specific case of surface-deployed (h = 0) coincident (b = a) coils,

$$\mathcal{M}(0) = \int_0^\infty \left[J_1(\lambda a) \right]^2 d\lambda = \infty.$$

Thus, in theory, for instantaneous source current termination, the impulsive response is infinite.¹ In practice, of course, the response is finite for finite termination time, finite standoff and practical coil configurations. The main finding, however, is that a nonconductive and non-dispersive magnetic soil yields no sustained secondary response. The same cannot be said for a dispersive or viscous magnetic soil.

Non-conductive, dispersive magnetic soil

As Das (2004) has demonstrated, a useful approximation for the response over a nonconductive, dispersive magnetic soil is obtained by generalizing the previous result for a frequencydependent magnetic susceptibility. In particular, a well established model for susceptibility dispersion of a viscous magnetic soil (Richter 1937; Chikazumi 1965; Lee 1984) assumes a log-uniform distribution of grain-volume related relaxation constants. The corresponding magnetic susceptibility is given by

$$\chi(f) = \chi_{dc} \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \ln\left(\frac{1 + i2\pi f \tau_2}{1 + i2\pi f \tau_1}\right) \right],$$
(23)

where τ_1 and τ_2 denote, respectively, lower and upper bandlimits on the time-constant distribution.² For $\ln \tau_1 = \ln \tau_{\chi} - \ln(\tau_2/\tau_1)/2$, $\ln \tau_2 = \ln \tau_{\chi} + \ln(\tau_2/\tau_1)/2$ and in the limit $\tau_2/\tau_1 \rightarrow 1$, equation (23) reduces to the standard Debye dispersion relation (Debye 1929)

$$\chi(f) = \frac{\chi_{dc}}{1 + i2\pi f \tau_{\chi}}$$
(24)

and the accompanying transient decay of magnetization is purely exponential. In general, for an arbitrary time-constant distribution $T_{\chi}(\tau)$, with $\int_{0}^{\infty} T_{\chi}(\tau) d\tau = 1$, we have

$$\chi(f) = \chi_{dc} \int_0^\infty \frac{\mathcal{T}_{\chi}(\tau)}{1 + i2\pi f \tau} d\tau, \qquad (25)$$

which yields Equation (23) for the specific $1/\tau$ -scaled logarithmic time-constant distribution

$$\mathcal{T}_{\chi}(\tau) = \mathcal{T}_{\chi}(\ln \tau) / \tau = \begin{cases} 0, & \tau < \tau_{1} \\ 1 / \tau \ln(\tau_{2} / \tau_{1}), & \tau_{1} \le \tau \le \tau_{2} \\ 0, & \tau > \tau_{2} \end{cases}$$
(26)

The nature of related frequency dependence is displayed in Fig. 4 for $\log(\tau_2/\tau_1)$ ranging between 0.0 (Debye, $\chi_{FD} \sim 90\%$)

¹In theory (equations (12) and (20)), the associated primary step response $v_p(t) = -\mu_0 \pi a^2 I \delta(t) \int_0^\infty [J_1(\lambda a)]^2 d\lambda$ is similarly impulsive and infinite for coincident coils and independent of stand-off (*b*).

²Note: $\ln x \equiv \log_e x$ and $\log x \equiv \log_{10} x$.

and 100.0 ($\chi_{FD} \sim 2\%$).³ In general, together with available empirical evidence (e.g., Dearing *et al.* 1996), the foregoing model suggests that a relatively broad time-constant bandwidth log(τ_2/τ_1) > 10 ($\chi_{FD} < 15 - 20\%$) is the practical reality. Indeed, it is well established (Nagata 1961; Dunlop and Özdemir 1997) that a relatively minute variation in the grain volume of magnetic particles (in proximity of the stable single-domain/superparamagnetic transition) is associated with a comparatively enormous swing in related viscous timeconstants.

Now, recognizing that for the vast majority of soils $\chi \ll$ 2, we have from equation (19)

$$1 + r_{\rm TE} \approx 1 + \frac{\chi\left(f\right)}{2} \tag{27}$$

and, therefore,

$$w(t) \approx -\mu_0 \pi a^2 I \mathcal{M}(0) \\ \int_{-\infty}^{\infty} \left(1 + \frac{\chi_{dc}}{2} \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \ln\left(\frac{1 + i2\pi f \tau_2}{1 + i2\pi f \tau_1}\right) \right] \right) e^{i2\pi f t} df$$
(28)

from equation (12). Finally, substituting $s = i2\pi f$ and applying standard Laplace transforms yields

$$v(t) \approx -\mu_0 \pi a^2 I \mathcal{M}(0) \left[\delta(t) - \frac{\chi_{dc}}{2 \ln(\tau_2/\tau_1)} \frac{1}{t} \left(e^{-t/\tau_1} - e^{-t/\tau_2} \right) \right]$$
(29)

and assuming $\tau_1 << \tau << \tau_2$,

$$v(t) \approx -\mu_0 \pi a^2 I \mathcal{M}(0) \frac{\chi_{dc}}{2 \ln (\tau_2/\tau_1)} \frac{1}{t}.$$
(30)

Note that in addition to an instantaneous primary impulse associated with source current termination, magnetic viscosity leads to a sustained transient response having t^{-1} dependence. Thus, in comparison with $t^{-5/2}$ late-time dependence due to eddy currents in a non-magnetic and non-dispersive conductive soil (equation (18)), signal decay is far more gradual for a viscous magnetic soil, resulting in a substantially enhanced and potentially anomalous response.

In particular, Das (2006) compared the late-time response predicted by equation (30) with that for a purely conductive soil as per equation (18). It was observed that even for an extreme soil conductivity of $\sigma_{dc} = 5$ S/m, a viscous magnetic soil with only moderate static susceptibility $\chi_{dc} = 0.0005$ SI

yielded a substantially greater response at times exceeding 10 μ s. We shall return to the case of a viscous magnetic soil in due course. First, we consider the more general case of a magnetic and conductive soil.

Non-dispersive, conductive-magnetic soil

Lee (1984) treated the common case of a soil having both finite electrical conductivity and magnetic susceptibility. Beginning with equation (1) and considering a single-turn coil of finite cross-sectional radius ϱ , a corresponding field relation was obtained for step source current with

$$1 + r_{\rm TE} = \frac{2\mu_1\lambda}{\mu_1\lambda + \mu_0u_1}.$$
 (31)

The corresponding time-domain response is

$$(t) = -2\mu_0 \pi a^2 I \int_0^\infty \int_A \int_{-\infty}^\infty \frac{2\mu_1 \lambda}{\mu_1 \lambda + \mu_0 u_1} \frac{e^{i2\pi ft}}{\pi \varrho^2} df$$
$$\times [J_1(\lambda a)]^2 dA d\lambda, \qquad (32)$$

where A represents the cross-sectional area of a coincident receiver loop (conductor).

Assuming a non-dispersive soil with $\mu_1 = 1 + \chi_{dc}$, the integral with respect to f is evaluated by appropriate contour integration and the integral over the cross-section of the coil is approximated by $\pi \varrho^2$. Finally, the integral with respect to λ is evaluated by expanding the Bessel functions as a power series and integrating term by term. For small χ_{dc} Lee obtained the following late-time $(t/\sigma \mu_0 a^2 \gg 1)$ response by Taylor series approximation about $\chi_{dc} = 0$ (see also Ignetik, Thio and Westfold 1985)

$$v(t) = v_{3}(t) \approx -\frac{\mu_{0}\sqrt{\pi}aI}{20t} \left(\frac{\sigma\mu_{0}a^{2}}{t}\right)^{3/2} - \frac{19\mu_{0}\chi_{dc}\sqrt{\pi}aI}{280t} \left(\frac{\sigma\mu_{0}a^{2}}{t}\right)^{3/2}.$$
(33)

Comparing this result with equation (18), it is observed that the first term is identical to the late-time response for a non-magnetic soil. Moreover, the second term associated with frequency-independent susceptibility χ_{dc} is characterized by an equivalent t^{-5/2} time dependence. Thus, as Lee suggested, the response can be rewritten as

$$v(t) \approx -\frac{\mu_0 \sqrt{\pi} a I}{20t} \left(\frac{\sigma_a \mu_0 a^2}{t}\right)^{3/2} = -\frac{\sqrt{\pi} \mu_0^{5/2} \sigma_a^{3/2} a^4 I}{20} t^{-5/2},$$
(34)

where

v

$$\sigma_a = \sigma \left(1 + \frac{19\chi_{dc}}{14} \right)^{2/3}.$$
(35)

³A commonly referenced measure of the frequency dependence of magnetic susceptibility is defined as $\chi_{FD} = [(\chi_{LF} - \chi_{HF}) / \chi_{LF}] \times 100$, where χ_{LF} and χ_{HF} denote low-frequency and high-frequency susceptibility measured at frequencies f_L and f_H , spanning a decade. See Appendix A for the connection between χ_{FD} and common dispersion models.



Figure 5 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v2 - non-magnetic, non-dispersive conductive soil (late-time).

v3 – non-dispersive magnetic, non-dispersive conductive soil.

Response depicted for $\sigma_{dc} = 0.001$, $\sigma_{dc} = 0.01$ and $\sigma_{dc} = 0.1$ S/m (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).

denotes a correspondingly enhanced apparent conductivity. In effect, the influence of non-dispersive susceptibility on the latetime response is indistinguishable from a marginal increase in electrical conductivity. Induced magnetization enhances the decaying inductive field by a factor $\mu_r = \mu_1/\mu_0 = 1 + \chi_{dc}$. As indicated by equation (35), however, the effect is minor for typical soil susceptibilities and the resulting late-time response does not deviate appreciably from that predicted by equation (18). The related influence is illustrated in Figs 5 and 6 for a representative range of χ_{dc} . As demonstrated in previous sections, the principal signature of non-dispersive magnetic susceptibility is an impulsive signal accompanying source current termination at t = 0.

Conductive, dispersive magnetic soil

More generally, Lee (1984) considered frequency-dependent magnetic permeability. In particular, assuming the susceptibility dispersion model in equation (23) and Fig. 4, we have the related soil magnetic permeability

$$\mu_{1} = \mu_{0} \left(1 + \chi_{dc} \left[1 - \frac{1}{\ln(\tau_{2}/\tau_{1})} \ln\left(\frac{1 + i2\pi f \tau_{2}}{1 + i2\pi f \tau_{1}}\right) \right] \right), \quad (36)$$

in equation (31). More specifically, Lee (1984) expanded equation (31) via the Taylor series about $(\mu_1 - \mu_0)/\mu_0$ and evaluated the resulting field equation via methods similar to

those employed for non-viscous permeability. The resulting late-stage $(t/\sigma \mu_0 a^2 \gg 1)$ response is

$$v(t) = v_4(t) \approx -\frac{\mu_0 \sqrt{\pi} a I}{20t} \left(\frac{\sigma_a \mu_0 a^2}{t}\right)^{3/2} + \frac{19\mu_0 \chi_{dc} \sqrt{\pi} a I}{280t} \left(\frac{\sigma \mu_0 a^2}{t}\right)^{3/2} \left[\frac{\psi(5/2)}{\ln(\tau_2/\tau_1)} - \frac{\ln(t/\tau_2)}{\ln(\tau_2/\tau_1)}\right], \\ - \frac{2\mu_0 \chi_{dc} \pi a I}{3t \ln(\tau_2/\tau_1)} \left(\frac{\sigma \mu_0 a^2}{t}\right) - \frac{\mu_0 \chi_{dc} a I}{2t} \frac{\ln(2a/\varrho)}{\ln(\tau_2/\tau_1)}$$
(37)

where the digamma function $\psi(5/2) \approx 0.703157$. Note that the initial term is consistent with equations (34)–(35), comprising the late-time response for a soil having apparent conductivity σ_a , including the influence of non-viscous magnetic susceptibility (for $\tau_2/\tau_1 \rightarrow \infty$, equation (37) reduces to equation (34)).⁴ The remaining terms reflect the influence of magnetic viscosity and related frequency-dependent permeability as described by equation (36). In general, for practical coil geometries, coil radius *a* is large compared to its cross-sectional radius ρ and the late-time response is ultimately dominated by the final term with t^{-1} dependence, compared with $t^{-5/2}$

⁴Related expressions (31) and (35) in Lee (1984) are not apparently consistent. In particular, Lee's equation (35) (for $\tau_1 \rightarrow 0$) appears to imply a marginally higher apparent conductivity $\sigma_a = \sigma (1 + 19\sqrt{\pi}\chi dc/14)^{2/3}$ than corresponding equation (31). Related deviation is indistinguishable on the scale of current diagrams.



Figure 6 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v2 - non-magnetic, non-dispersive conductive soil (late-time).

v3 – non-dispersive magnetic, non-dispersive conductive soil. Response depicted for $\chi_{dc} = 0.00001$ and $\chi_{dc} = 0.1$ SI (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).

dependence for non-dispersive soils. Thus, as previously discussed, magnetic viscosity yields a substantially enhanced and potentially anomalous response. The value of a/ρ , combined with the electrical conductivity σ_{dc} , magnetic susceptibility χ_{dc} and viscous magnetic time-constants τ_1 , τ_2 , determine the precise transition from $t^{-5/2}$ to t^{-1} dependence.

The predicted response is displayed in Fig. 7 for $\chi_{dc} = 0.001$, log $(\tau_2/\tau_1) = 20$ $(\chi_{FD} \approx 10\%)$, $a/\varrho = 100$ and the same three conductivity values in Fig. 5 and previous figures. Decreasing the ratio a/ϱ by a factor of 10 only marginally reduces the predicted response. In general, with increasing electrical conductivity, predicted transition from $t^{-5/2}$ to t^{-1} dependence is effectively delayed as per equation (37). However, assuming typical coil dimensions for hand-held systems and practical limits on soil electrical conductivity, the viscous magnetic response is generally predominant within the practical measurement range ($t > 10 \ \mu s$). We shall return to this issue in following discussion. First, we consider the relative influence of specific magnetic parameters.

While magnetic dispersion is an essential condition for sustained t^{-1} decay, Fig. 8 illustrates that the predicted response is not as sensitive to the degree of underlying, intrinsic dispersion as it is to the absolute level of magnetic susceptibility. In particular, Fig. 8(a) displays the influence of increasing the time-constant bandwidth from log (τ_2/τ_1) = 10 ($\chi_{FD} > 15\%$) to log $(\tau_2/\tau_1) = 100$ ($\chi_{FD} \approx 2\%$), a representative range for the majority of magnetic soils that are nominally viscous. It is observed that the corresponding order of magnitude reduction in the related response is relatively minor compared with the effect of varying the static (low-frequency) susceptibility χ_{dc} over a representative range spanning several orders of magnitude for a nominal time-constant bandwidth (Fig. 8b). Thus, although magnetic dispersion is a prerequisite condition for the critical transition from $t^{-5/2}$ to t^{-1} decay rate, the onset and level of the viscosity dominated response is more strongly influenced by absolute magnetic susceptibility. In effect, absolute susceptibility scales underlying dispersion to yield net frequency dependence (viscosity) that can be considerable despite a relatively modest χ_{FD} . Indeed, this explains the direct influence of both absolute susceptibility and related frequency dependence, or differential susceptibility ($\Delta \chi = \chi_{LF} - \chi_{HF}$), on ground reference height (GRH) for a standard calibrated metal detector as reported by Guelle et al. (2006).

Now, returning to equation (30), it is useful to compare the late-time approximation

$$v(t) = v_5(t) \approx -\mu_0 \pi a^2 I \mathcal{M}(0) \frac{\chi_{dc}}{2 \ln(\tau_2/\tau_1)} \frac{1}{t}$$
(38)

with the foregoing result (equation (37)).

Note that in place of $\mathcal{M}(0) = \infty$ as predicted by equation (22) for b = a, b = 0, equation (37) implies $\mathcal{M}(0) = \ln(2a/\varrho)/\pi a$. The resulting approximation is illustrated in Fig. 9. As expected, there is excellent agreement with the predicted t^{-1} response over the practical measurement range (10 μ s - 1000 μ s). Again, although equation (38) hinges on dispersive magnetic susceptibility, the expression also emphasizes the scaling and potentially predominant influence of absolute susceptibility χ_{dc} (largely reflecting the composition and concentration of the viscous soil magnetic fraction).

Equating expressions (38) and (34), we obtain the following approximate relation

$$t_{\rm V} \approx \mu_0 \sigma_a a^2 \left[\frac{\sqrt{\pi}}{10 \chi_{dc}} \frac{\ln \left(\tau_2 / \tau_1\right)}{\ln \left(2a / \varrho\right)} \right]^{2/3} \tag{39}$$

for the transition time from $t^{-5/2}$ to t^{-1} dependence for a conductive and viscous magnetic soil. Although the resulting expression inevitably overestimates the actual transition time,



Figure 7 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v1 - non-magnetic, non-dispersive conductive soil.

v2 - non-magnetic, non-dispersive conductive soil (late-time).

v3 - non-dispersive magnetic, non-dispersive conductive soil.

v4 - dispersive magnetic, non-dispersive conductive soil.

Note t^{-1} late-time dependence for the dispersive magnetic response. Response depicted for $\sigma_{dc} = 0.001$, $\sigma_{dc} = 0.01$ and $\sigma_{dc} = 0.1$ S/m (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).



Figure 8 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v1 - non-magnetic, non-dispersive conductive soil.

v2 – non-magnetic, non-dispersive conductive soil (late-time).

v3 - non-dispersive magnetic, non-dispersive conductive soil.

v4 - dispersive magnetic, non-dispersive conductive soil.

Response depicted for $\tau_2/\tau_1 = 10^{10}$, $\tau_2/\tau_1 = 10^{100}$, $\chi_{dc} = 0.00001$ and $\chi_{dc} = 0.1$ SI (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).



Figure 9 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v2 - non-magnetic, non-dispersive conductive soil (late-time).

v3 – non-dispersive magnetic, non-dispersive conductive soil.

v4 - dispersive magnetic, non-dispersive conductive soil.

v5 - dispersive magnetic, non-dispersive conductive soil (late-time).

Response depicted for $\sigma_{dc} = 0.001$, $\sigma_{dc} = 0.01$ and $\sigma_{dc} = 0.1$ S/m (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).

it is an adequate approximation and provides additional insight into the related influence of specific soil electromagnetic parameters. In particular, Fig. 10 illustrates the offsetting in-



Figure 10 Viscous magnetic transition time t_V as a function of static magnetic susceptibility χ_{dc} and time-constant bandwidth log (τ_2/τ_1) (open symbols for log $(\tau_2/\tau_1) < 10$). The dashed line indicates specific t_V for nominal soil model parameters as indicated.

fluence of the time-constant bandwidth log (τ_2/τ_1) and absolute low-frequency susceptibility χ_{dc} .

Given well established bounds on χ_{dc} and accepting log $(\tau_2/\tau_1) > 10$ $(\chi_{FD} < 15 - 20\%)$ as an empirical lower limit, it remains of interest to consider a practical upper limit on the time-constant bandwidth and related implications. In theory, so long as $\tau_2/\tau_1 \ge 1$ remains finite, the corresponding late-time response continues to be viscosity dominated and displays t^{-1} dependence for $t \ge t_V$. Obviously, however, as log (τ_2/τ_1) increases, a proportional rise in χ_{dc} is required to maintain a given t_V and related signal level. Consequently, effective magnetic viscosity $\nu \approx \chi_{dc}/\log(\tau_2/\tau_1)$ is the preferred parameter⁵ for gauging the net influence of soil magnetic susceptibility and related viscosity.

⁵As a practical matter, magnetic viscosity is estimated as $\nu \approx \Delta \chi / \log(f_H/f_L)$, with $\Delta \chi = \chi_{LF} - \chi_{HF}$ denoting the differential susceptibility measured between measurement frequencies f_L and f_H (see equation (52)). In particular, for measurement frequencies spanning a decade $(f_H/f_L = 10)$, $\nu \approx \Delta \chi$. In contrast and owing to normalization by χ_{LF} , related parameter $\chi_{FD} = (\Delta \chi / \chi_{LF}) \times 100$ provides improved discrimination of intrinsic dispersion related to the grain-size distribution of soil magnetic content, but fails to reflect the net extent of effective magnetic viscosity.



Figure 11 Viscous magnetic transition time t_V as a function of coil radius *a* with static conductivity σ_{dc} as the parameter (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).

Equation (39) also indicates that transition time t_V scales with the coil radius in approximate accordance with a standard quasi-static scaling relation (e.g., Frischknecht 1987)

$$t_{\rm V}' \approx t_{\rm V} \left(\frac{a'}{a}\right)^2,\tag{40}$$

where $t'_{\rm V}$ denotes the viscous transition time for scaled coil radius *a'*. Figure 11 displays $t_{\rm V}$ as a function of coincident coil radius *a*, with low-frequency conductivity σ_{dc} as a parameter. Trendlines are fit over the range a = 0.1 - 1.0 m. The deviation (reduction) from standard squared dependence on *a'*/*a* increases as the ratio *a*/ ρ increases with decreasing coil radius. As previously noted, however, the influence is subtle.

The corresponding scaling relation for conductivity ($\sigma = \sigma_a \approx \sigma_{dc}$) is

$$t'_{V} = t_{V} \left(\frac{\sigma'_{dc}}{\sigma_{dc}} \right). \tag{41}$$

Non-magnetic, dispersive conductive soil

Finally and of particular interest here, Lee (1981) and El-Kailouby *et al.* (1995, 1997) investigated the influence of frequency-dependent electrical conductivity on the TDEM response of a surface-deployed coincident-coil system. Assuming a non-magnetic soil, we have as per equations (4) and (13)

$$1 + r_{\rm TE} = 1 + \frac{\lambda - u_1}{\lambda + u_1}$$
(42)

with $u_1 = [\lambda^2 + i2\pi f \mu_0 \sigma(f)]^{1/2}$. Substituting equation (42) in equation (12) and ignoring the impulsive primary response

associated with termination of the source current at t = 0, yields the step response due to soil

$$v(t) = \mu_0 \pi a^2 I \int_0^\infty \int_{-\infty}^\infty \frac{\lambda - u_1}{\lambda + u_1} \left[J_1(\lambda a) \right]^2 d\lambda e^{i2\pi f t} df.$$
(43)

Lee (1981) evaluated the integral with respect to λ , obtaining a power series solution that is subsequently integrated with respect to complex frequency $f = Re^{i\psi}/2\pi$ via contour integration.

Assuming a Cole-Cole model for electrical conductivity dispersion (Cole and Cole 1941; Pelton *et al.* 1978)

$$\sigma(f) = \sigma_{dc} \left[1 + m \left(\frac{\left[i2\pi f \tau_{\sigma}\right]^c}{1 + \left[1 - m\right] \left[i2\pi f \tau_{\sigma}\right]^c} \right) \right],\tag{44}$$

the resulting asymptotic integral solution is

$$v(t) = v_6(t) = -\mu_0 a I \int_0^\infty e^{-Rt \sin \psi} \Sigma(R) dR,$$
 (45)

where

$$\Sigma(R) = \sum_{n=0}^{\infty} \left(\frac{4 \cos \alpha_n [a F(R)]^{2n+3} (2n+2)!}{(2n+5)! (n+1)! n!} - \frac{2 \cos \beta_n [a F(R)]^{2n+2} 2^{4n+2} n! n!}{\pi (2n+4)! (2n)!} \right),$$
(46)

R is the contour integration variable and angle $\psi = \pi/4$ defines the branch cut geometry. In addition,

$$F(R) = \left[\mu_0 \sigma_{dc} R \left(\frac{1 + 2\delta + \gamma^2}{1 + 2(1 - m)\delta + (1 - m)^2 \gamma^2} \right)^{1/2} \right]^{1/2},$$

$$\alpha_n = Rt \sin(\pi/2 + \psi) + \psi + (2n + 3)(\pi/2 + \psi + \phi)/2,$$

$$\beta_n = Rt \sin(\pi/2 + \psi) + \psi + (2n + 2)(\pi/2 + \psi + \phi)/2,$$

with

$$\begin{split} \gamma &= (\tau_{\sigma} R)^{c}, \\ \delta &= \gamma \cos\left[\left(\pi/2 + \psi\right) c\right], \\ \eta &= \gamma \sin\left[\left(\pi/2 + \psi\right) c\right], \\ \phi &= \tan^{-1}\left[\frac{\eta}{1+\delta}\right] - \tan^{-1}\left[\frac{(1-m)\eta}{1+(1-m)\delta}\right]. \end{split}$$

With change of integration variable $r = Rt \sin \psi$, the response is readily evaluated numerically employing a Gauss-Laguerre quadrature formulation (El-Kailouby *et al.* 1995). Results presented by Lee (1981) for specific Cole-Cole parameters, were confirmed independently by Raiche (1983) by way of an alternative computational method. As a check on our numerical evaluation of equation (45) the same results were reproduced prior to investigating response characteristics on



Figure 12 Cole-Cole complex electrical conductivity model. Normalized real σ'/σ_{dc} and imaginary σ''/σ_{dc} components are charted as functions of normalized frequency $2\pi f \tau_{\sigma}$, with Cole-Cole distribution constant *c* as the parameter.

the scale of hand-held sensors. In general, the response predicted by equation (45) is characterized by late-time polarity reversal and a diminished decay rate. Specific characteristics depend on the nature and degree of electrical dispersion.

In contrast with soil magnetic dispersion that is largely determined by the grain-size distribution of mineral magnetic content, soil electrical dispersion affecting TDEM sensors is due to a wide range of polarization and electrochemical processes that are influenced by an equally broad range of soil physicochemical parameters (De Loor 1983; Olhoeft 1985). Related relaxation/reaction rates range over several orders of magnitude with associated time-constant distributions analogous to those for magnetic dispersion.

Frequency dependence of the electrical conductivity as specified by equation (44) depends on the static value σ_{dc} , the corresponding high-frequency value σ_{∞} (or chargeability $m = (\sigma_{\infty} - \sigma_{dc})/\sigma_{\infty}$), the Cole-Cole distribution parameter c and reference time-constant τ_{σ} . as illustrated in Fig. 12.

In particular, for c = 1.0, equation (44) yields the related Debye-like dispersion relation

$$\sigma(f) = \sigma_{\infty} - \frac{\sigma_{\infty} - \sigma_{dc}}{1 + (1 - m)i2\pi f\tau_{\sigma}}$$
(47)

and the accompanying transient decay of induced polarization is exponential. For an arbitrary time constant distribution $T_{\sigma}(\tau)$, with $\int_{0}^{\infty} T_{\sigma}(\tau) d\tau = 1$, we obtain the general relation.

$$\sigma(f) = \sigma_{\infty} - (\sigma_{\infty} - \sigma_{dc}) \int_0^\infty \frac{\mathcal{T}_{\sigma}(\tau)}{1 + (1 - m)i2\pi f \tau} d\tau.$$
(48)

A $1/\tau$ -scaled, logarithmic time-constant distribution having form

$$\mathcal{T}_{\sigma}(\tau) = \mathcal{T}_{\sigma}(\ln \tau)/\tau = \frac{1}{2\pi\tau} \frac{\sin\left[(1-c)\pi\right]}{\cosh\left[c \ \ln(\tau/\tau_{\sigma})\right] - \cos\left[(1-c)\pi\right]}$$
(49)

yields the Cole-Cole dispersion relation of equation (44). Figure 12 displays the predicted normalized frequency dependence for $\sigma_{dc} = .01 \text{ mS/m}$, m = 0.3, $\tau_0 = 0.0001 \text{ s}$ and for the Cole-Cole distribution parameter *c* ranging between 0.01–1.0.

The late-time response predicted by equation (45) for a nonmagnetic, electrically polarizable soil is depicted in Fig. 13 (together with previous models) for three values of static conductivity. Note that predicted polarity reversal occurs between $0.1-1.0 \ \mu$ s and that corresponding later-time decay rates are substantially reduced compared with the $t^{-5/2}$ dependence observed for non-dispersive conductive soil. As Lee (1981) noted for $t/\sigma \mu_0 a^2 > 10$, the corresponding late-time response is well approximated by retaining only the initial (n = 0) term of the asymptotic series of equation (46). For parameters in Fig. 13, the deviation between single-term and multi-term approximations is minimal for $t \ge 1.0 \ \mu$ s.

For the specific case $\sigma_{dc} = 0.01$ S/m, Fig. 14 provides some indication of the influence of individual Cole-Cole parameters on the predicted response. For all but the most rapidly acting polarization processes ($\tau_{\sigma} > 10^{-8}$ s), polarity reversal and related decay rate reduction occur later as the associated time-constant τ_{σ} increases (Fig. 14a). Increasing chargeability *m* has, in general, the opposite effect (Fig. 14b), leading to earlier polarity reversal. Dependence on the Cole-Cole



Figure 13 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v1 – non-magnetic, non-dispersive conductive soil.

v2 - non-magnetic, non-dispersive conductive soil (late-time).

v3 - non-dispersive magnetic, non-dispersive conductive soil.

v4 - dispersive magnetic, non-dispersive conductive soil.

v6-p - non-magnetic, dispersive conductive soil ('positive' (normal) polarity).

v6-n - non-magnetic, dispersive conductive soil ('negative' (reverse) polarity).

Response depicted for $\sigma_{dc} = 0.001$, $\sigma_{dc} = 0.01$ and $\sigma_{dc} = 0.1$ S/m (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).



Figure 14 TDEM response for a horizontal, coincident-coil system on a soil half-space.

v1 - non-magnetic, non-dispersive conductive soil.

v2 – non-magnetic, non-dispersive conductive soil (late-time).

v3 - non-dispersive magnetic, non-dispersive conductive soil.

v4 - dispersive magnetic, non-dispersive conductive soil.

v6-p - non-magnetic, dispersive conductive soil ('positive' (normal) polarity).

v6-n - non-magnetic, dispersive conductive soil ('negative' (reverse) polarity).

Response depicted for $\tau_{\sigma} = 10^{-2}$, $\tau_{\sigma} = 10^{-6}$, m = 0.1, m = 0.7, c = 0.05 and c = 0.2 (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).



Figure 15 Polarity reversal time t_R as a function of Cole-Cole conductivity dispersion parameters, including reference time-constant τ_{σ} , distribution parameter *c* and chargeability *m*. The dashed line indicates t_R for nominal soil model parameters as indicated.

distribution parameter c is more complicated. For specific parameter values identified in Fig. 14(c), polarity reversal occurs at minimum time for $c \approx 0.2$ and increases for both lesser and greater values. The general nature and relative influence of Cole-Cole parameters on polarity reversal time t_R are illustrated in Fig. 15 for representative parameter ranges and reference values as per Fig. 14.

The corresponding rate of increase in related decay rates, following peak negative-valued response is also significant. Related variation with Cole-Cole parameter values is displayed in Fig. 16. In general, it is observed that polarization parameters have considerable and complicated influence over the transition from polarity reversal to late-time $t^{-5/2}$ dependence. In particular, for specific parameter values considered, Fig. 16 suggests that sustained response (reduced decay rate) immediately following polarity reversal is associated with increasing τ_{σ} , moderate *m* and increasing *c*. It is also observed, however, that the same characteristics do not generally lead to earlier phase reversal.

Finally, it is important to appreciate that response characteristics are also dependent on coil radius and conductivity, with polarity reversal occurring later as the product of coil radius and conductivity increases. Figure 17 displays polarity reversal time t_R as a function of coincident coil radius *a*, with low-frequency conductivity σ_{dc} as a parameter. Lower curves are for c = 0.5 and upper curves for c = 0 with remaining parameters as per the reference model in Fig. 14. Results imply a generalized scaling relation

$$t_R' = t_R \left(\frac{a'}{a}\right)^{2(1-c)},\tag{50}$$

where, t'_R denotes the reversal time for scaled coil radius a'. Note that for an infinitely broad time-constant distribution (c = 0), equation (50) reduces to the standard quasistatic relation $t'_R = t_R(a/a')^2$ with squared dimensional dependence (e.g., Frischknecht 1987) as equation (45) reverts to equations (15)–(18) for non-polarizable soil with low- frequency conductivity $\sigma = 2\sigma_{dc}/(2 - m)$ (Lee 1981). The corresponding scaling relation for conductivity follows as

$$t'_{R} = t_{R} \left(\frac{\sigma'_{dc}}{\sigma_{dc}}\right)^{1-c}.$$
(51)

The principal finding, however, is that induced electrical polarization produces signal polarity reversal and related



Figure 16 Influence of Cole-Cole conductivity dispersion parameters on the related TDEM decay rate following polarity-reversal. The predicted time dependence of TDEM response charted as a function of time with reference time-constant τ_{α} , chargeability m and distribution constant *c* as parameters (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).



Figure 17 Polarity reversal time t_R as a function of coil radius *a* with static conductivity σ_{dc} as a parameter and for two values of the Cole-Cole time-constant distribution parameter c = 0.0 and c = 0.5 (coincident coil radius a = b = 0.1 m and nominal soil model parameters as indicated).

decay rate reduction within the typical measurement range for metal detectors and related small-scale TDEM sensors. Moreover, it is evident on the basis of theory-predicted results in foregoing Figs 8, 13 and 14 that specific soil electromagnetic parameters can potentially combine to yield an effective response that is dominated by electrical polarization despite limited magnetic dispersion.

SUMMARY

The performance of pulse induction metal detectors and related hand-held sensors is significantly influenced by soil electromagnetic properties. However, in contrast with frequencydomain (FDEM) systems that are principally affected by absolute levels of soil magnetic susceptibility and electrical conductivity, TDEM systems are sensitive to related dispersion and associated viscosity.

In particular, where soils incorporate a substantial magnetic component having viscous susceptibility, the late-time response is generally enhanced with t^{-1} decay rate, compared with $t^{-5/2}$ for a non-magnetic or non-viscous soil.

With few exceptions, the model of soil magnetic dispersion described by equations (23) and (36) and leading to equations (37) and (38) (t^{-1} response), appears to be generally supported

by available data.⁶ In particular, recent multi-frequency measurements of soil magnetic susceptibility (West and Bailey 2005; Preetz and Igel 2005; Cross 2008) are largely consistent with following approximate relations (for $1/2\pi\tau_2 << f << 1/2\pi\tau_1$, $\tau_1 << t << \tau_2$)

$$\frac{\chi_{dc}}{\log (\tau_2/\tau_1)} = \frac{\partial \chi'(f)}{\partial \log f} = \frac{2\chi''(f)}{\pi \log e} = -\frac{1}{H_0} \frac{\partial M(t)}{\partial \log t} = v \quad (52)$$

(Mullins and Tite 1973; Dabas, Jolivet and Tabbagh 1992), where the right-most relation defines magnetic viscosity v as the rate of change of time-dependent magnetization M(t) normalized by primary source field H_0 .

The general validity of foregoing relations and wellestablished empirical limits on χ_{FD} reflect natural grain-size variation and imply a correspondingly broad time-constant distribution (log (τ_2/τ_1) > 10). It is also observed that the net magnitude of soil magnetic viscosity depends on both the extent of underlying dispersion (grain-size/time-constant distribution) and the absolute susceptibility (composition and concentration) of soil magnetic content (i.e., $\nu_c \approx \chi_{dc}/\log(\tau_2/\tau_1)$).

Significantly, while the two factors are commonly related, it is decidedly more common to encounter a soil having elevated magnetic susceptibility and limited viscosity, than a substantially viscous soil with low susceptibility. The explanation is that substantial viscosity ($\chi_{FD} > 2\%$) is largely attributable to a significant fine-grained singledomain/superparamagnetic fraction that generally carries an anomalously elevated intrinsic magnetic susceptibility (Maher 1988; Forster, Evans and Heller 1994; Dearing *et al.* 1996). In contrast, a soil incorporating a substantial concentration of stable single-domain or multi-domain magnetic material can possess considerable magnetic susceptibility with limited or negligible viscosity.

In theory, only finite magnetic dispersion is required to produce the sustained and characteristic t^{-1} response. However, where magnetic dispersion is limited ($\chi_{FD} < 2\%$), related significance is increasingly dependent on the level of associated susceptibility (i.e., concentration of related viscous magnetic content) to support the related signature. Where viscous magnetic content is limited and related susceptibility is low, there is increasing potential for the associated signature to be overshadowed by the background $t^{-5/2}$ conductive response (i.e., t_V , given by equation (39) exceeds the effective measurement time). Clearly, in the limiting case of non-dispersive magnetic susceptibility, no amount of associated magnetic content yields t^{-1} dependence and whatever level of susceptibility only

⁶Dabas *et al.* (1992) note that a limited number of samples display significantly different response characteristics.

produces an effective enhancement of electrical conductivity and related response as per equations (34)–(35).

Notably, findings also demonstrate that appreciable dispersion of soil electrical conductivity can produce anomalous and sustained response characteristics within the typical measurement range of hand-held TDEM systems. In particular, significant electrical chargeability yields characteristic signal polarity reversal and time-dependent decay rate reduction. Numerical modelling on the basis of equations (45)–(46) predicts $t^{-3/2} - t^{-5/2}$ dependence. Most significantly, the results suggest that for soils with sufficient chargeability, related induced polarization could potentially dominate the TDEM response for soils having limited magnetic viscosity.

In practice, however and on the basis of limited available data (e.g., Ogilvy and Kuzmina, 1972; Mehran and Arulanandan 1977; Iliceto, Santarato and Veronese 1982; Olhoeft 1985, 1987) it is anticipated that the requisite level of electrical conductivity dispersion is relatively rare. Assuming (as a minimum condition) that predicted reversal time t_R must precede viscous transition time t_V (equation (39)), comparison of Figs 10 and 15 indicates that for nominal model parameters, required overlap is limited. However, the potential does exist, predictably for low magnetic viscosity (large $\log(\tau^2 / \tau^1)$) with low susceptibility (small χ_{dc}), and for high electrical chargeability (large *m*, large *c*) with high conductivity (large σ_{dc}).

Additional work is required to characterize the nature and range of electrical conductivity dispersion in soils. In particular, it is noted that much of available data is based on gated time-domain (galvanic) chargeability measurements and could potentially underestimate the full extent of chargeability for TDEM sensors. Future measurements should focus on fullwaveform or frequency-domain analysis of soils known to be problematic for pulsed-induction metal detectors and with simultaneous characterization of magnetic properties. Preliminary investigation of three such soils (Cross 2008) appears to confirm magnetic viscosity as the key parameter with relatively limited indication of significant electrical dispersion. However, a more extensive study is obviously required to yield broadly meaningful and representative conclusions.

CONCLUSIONS

While the principal role of magnetic viscosity in limiting the performance of hand-held time domain electromagnetic sensors is well established and confirmed by the present study, it is also demonstrated that significant electrical dispersion can lead to anomalous response characteristics within equivalent time gates. Together with laboratory and *in situ* measurements of frequency-dependent soil electromagnetic properties, the foregoing theoretical framework provides an initial indication of the nature and relative extent of related soil influence and potential implications for metal detector performance.

In practice, real-world soil heterogeneity and related spatial variability of soil electromagnetic properties also give rise to localized fluctuation of the background response or 'soil noise' that can potentially emulate or mask the signatures of landmines, UXO or other targets of interest. Ultimately, a fuller evaluation of soil influence on metal detector performance will require three-dimensional numerical modelling and related statistical analysis for realistic soil electromagnetic property distributions.

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APPENDIX A

χ_{FD} related to common dispersion models

In connection with measurement and characterization of soil magnetic properties, the frequency dependence of magnetic susceptibility is commonly quantified via the parameter

$$\chi_{FD} = [(\chi_{LF} - \chi_{HF})/\chi_{LF}] \times 100,$$
 (A1)

where χ_{LF} and χ_{HF} denote low-frequency and high-frequency susceptibility values, respectively and where related measurement frequencies are separated by a decade. In particular, χ_{LF} and χ_{HF} refer to the real-valued (in-phase) component χ' of the complex, frequency-dependent magnetic susceptibility $\chi(f) = \chi(f) - i\chi''(f)$ measured at frequencies $f_L = 465$ Hz and $f_H = 4.65$ kHz, respectively, as employed by the de facto standard Bartington MS2B susceptibility meter.

Figure A1 displays the relationship between χ_{FD} and three well-established dispersion models. In addition to the Chikazumi (originally Richter 1937 and also Frölich 1958) model, associated with a log-uniform distribution of magnetic time constants

$$\mathcal{T}_{\chi}(\ln \tau) = \begin{cases} 0, & \tau < \tau_{1}; \\ 1/\ln(\tau_{2}/\tau_{1}), & \tau \leq \tau \leq \tau_{2}, \\ 0, & \tau > \tau_{2} \end{cases}$$
(A2)

relations are also depicted for a more natural log-normal timeconstant distribution (Wagner 1913)

$$\mathcal{T}_{\chi}(\ln \tau) = \frac{b}{\sqrt{\pi}} \exp\left[-b^2 \ln^2 (\tau/\tau_{\chi})\right],\tag{A3}$$

and the common Cole-Cole relation (Cole and Cole 1941), having effective logarithmic time-constant distribution

$$\mathcal{T}_{\chi}(\ln \tau) = \frac{1}{2\pi} \frac{\sin [(1-c)\pi]}{\cosh [c \ln(\tau/\tau_{\chi})] - \cos[(1-c)\pi]}.$$
 (A4)

In all cases, the time-constant distribution is symmetric about a reference time-constant τ_{χ} , with breadth described by related distribution parameters $\log(\tau_2/\tau_1)$, *b* and *c*, respectively. Moreover, it is assumed for purposes of the present analysis that $\tau_{\chi} = 1/2\pi \bar{f}$, where $\bar{f} = 10^{(\log f_L + \log f_H)/2}$ denotes the log-distributed mean measurement frequency (i.e., for f_L =



Figure A1 Magnetic susceptibility dispersion parameter χ_{FD} as a function of time-constant distribution parameters log (τ_2/τ_1) (log-uniform), b^{-1} (log-normal) and *c* (Cole-Cole). The dashed lines link corresponding parameter values for specific cases $\chi_{FD} = 56.1\%$ and $\chi_{FD} = 32.4\%$ with corresponding time-constant and complex susceptibility spectra displayed in Figs A2 and A3, respectively.

465 Hz and $f_H = 4.65$ kHz, $\bar{f} \approx 1.47$ kHz and $\tau_{\chi} = 1/2\pi \ \bar{f} \approx 1.08 \times 10^{-4}$ s).

In general, the corresponding complex magnetic susceptibility follows from equation (25), with $T_{\chi}(\tau) = T_{\chi}(\ln \tau)/\tau$. However, analytical evaluation is intractable for the log-normal distribution and, consequently, discrete values of χ_{FD} as a function of *b* are approximated on the basis of numerical results reported by Yager (1936) (see also Nowick and Berry 1961).

As indicated by Fig. A1, χ_{FD} approaches a theoretical limit of approximately 90% as the distribution parameters approach log (τ_2/τ_1) = 0, 1/b = 0, c = 1.0 and the related complex susceptibility spectra approach the Debye spectrum for reference time-constant τ_{χ} (see Fig. 4). As the breadth of the respective time-constant distributions increases, χ_{FD} decreases in a non-linear fashion (e.g., $\chi_{FD} \approx [\ln(10)/\chi_{LF} \log(\tau_2/\tau_1)] \times$ 100 for $\log(\tau_2/\tau_1) > \sim 3$).



Figure A2 Time-constant distributions described by equations (A2)–(A4) for log-uniform, log-normal and Cole-Cole models, respectively. Related distributions for specific cases $\chi_{FD} = 56.1\%$ and $\chi_{FD} = 32.4\%$, with related parameters as indicated and identified in Fig. A1.

For sake of illustration, Figs A2 and A3 display model time-constant distributions and related susceptibility spectra for exaggerated χ_{FD} values of 56.1% and 32.4%, and for corresponding distribution parameters derived from Fig. A1. In general, results demonstrate that as the relaxation time-constant distribution becomes broader, related dispersion of the corresponding magnetic susceptibility decreases. Modelling also illustrates the idealized nature of the log-uniform (Richter) time-constant model compared with the more natural log-normal (Wagner) distribution.^{A1} Moreover, it is observed that the Cole-Cole model provides a useful approximation to the lognormal distribution with the advantage of an associated analytical expression for corresponding complex susceptibility.



Figure A3 Complex magnetic susceptibility spectra predicted for time-constant distributions in Fig. A2 for specific cases $\chi_{FD} = 56.1\%$ and $\chi_{FD} = 32.4\%$ and related distribution parameters as indicated. Dashed lines define the range bounded by normalized measurement frequencies $2\pi f L \tau_{\chi}$ and $2\pi f H \tau_{\chi}$.

With regards to χ_{FD} , results confirm that for distribution parameters indicated by Fig. A1, susceptibility spectra (particularly χ') arising for all three time-constant models are in good agreement over the decade of frequency centred on $\bar{f} = 1/2\pi\tau_{\chi}$ and yield consistent estimates of χ_{FD} . In practice, the upper limit on χ_{FD} for soils is commonly accepted to be roughly 15–20% (Dearing *et al.* 1996; Eyre 1997; Worm 1998; Muxworthy 2001) and, thus in theory, related magnetic time-constant distributions for natural soils are considerably broader than depicted in Fig. A2, with corresponding approximate parameter values $\log(\tau_2/\tau_1) > 10.0$, 1/b > 12.5(via extrapolation on values reported by Yager, 1936) and c < 0.17.

That the extent of soil magnetic viscosity is generally limited and that associated time-constant distributions are correspondingly broad are reflections of inevitable natural variability in the dimensions and geometry of constituent magnetic grains and the nature of thermally activated demagnetization. Because the related relaxation time-constant for a

^{A1}It is noted that the corresponding grain volume distribution is more fundamental and that related assumption of a log-normal volume distribution implies a corresponding time-constant distribution that is skewed toward larger relaxation times

single-domain grain is exponentially dependent on the ratio of volume to temperature, even a relatively narrow distribution of grain volumes yields a comparatively enormous span in the corresponding time-constant distribution (see Nagata 1961; Dunlop and Özdemir, 1997).

With regards to the influence of soil magnetic dispersion on TDEM sensors and related assessment, the principal implication is that log-linear susceptibility dispersion is the general expectation and that standard dual-frequency measurements are likely sufficient for the bulk of natural soils at typical operating frequencies. However, to emphasize the limitations of the foregoing analysis and to provide additional insight on χ_{FD} as a measure of magnetic dispersion, we offer the following further observations.

As previously noted, the susceptibility spectra in Fig. A3 are centred with respect to standard measurement frequencies by referring associated time-constant distributions (Fig. A2) to relaxation time $\tau_{\chi} = 1/2\pi \tilde{f}$. In general, shifting the time-constant distribution toward lesser time-constants yields an effective decrease in χ_{FD} , initially due to normalization by larger χ_{LF} and subsequently to increasing non-linearity of the nominally log-linear spectrum. A similar shift toward larger

time-constants has the opposite effect. Moreover, for timeconstant distributions bracketing τ_{χ} and having sufficient bandwidth, it is notable that a lower limiting time-constant τ_1 has controlling influence on χ_{FD} . In effect, χ_{FD} fails to reflect the full extent of viscosity associated with larger grain volumes and correspondingly longer relaxation times. Finally, while Fig. A1 implies that χ_{FD} ultimately vanishes with infinite time-constant bandwidth, it should be appreciated that the intrinsic atomic reorganization interval, $\tau_0 > 10^{-12}$ (Dearing *et al.* 1996; Worm 1998), imposes a practical lower limit on τ_1 and, in turn, on χ_{FD} .

Consequently, in addition to a broad time-constant distribution, other considerations including the volumedependence of stable single-domain and superparamagnetic susceptibility, micro-coercivity variation, magnetic grain interaction and the influence of non-dispersive magnetic fractions are required to explain the full range of empirical χ_{FD} values (Forster *et al.* 1994; Dearing *et al.* 1996; Eyre 1997; Worm 1998; Muxworthy 2001)

For the assessment of soil magnetic influence on TDEM sensors, however, it is emphasized that magnetic viscosity $\nu \approx (\chi_{LF} - \chi_{HF})/\log(f_H/f_L)$ is the preferred parameter.